Inductive Reasoning

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Definition

Inductive reasoning is a logical process in which multiple premises, all believed true or found true most of the time, are combined to obtain a specific conclusion or to supply evidence for the truth of a conclusion. Inductive reasoning is often used to generate predictions or to make forecasts. Inductive reasoning differs from deductive reasoning in that while the conclusion of a deductive inference is certain, the truth of the conclusion of an inductive inference is only probable, where the degree of certainty is based upon the strength (or consistency) of the evidence. In other words, the conclusion of an inductive inference is not a logical certainty (such as when a meteorologist predicts snow). Inductive reasoning also encompasses most cases of where a general principle is derived or where categories are formed based on specific observations (provided that they are probabilistic in nature).

In practice, inductive reasoning is the logical foundation of science, and all fields (from physics to sociology) share the inductive method at their core. (In this regard, it is worth noting that, unlike mathematicians and logicians, scientists can never be 100% certain about their "truths" and can only make approximations based on accumulated empirical evidence.) But the application of inductive reasoning goes much beyond science and is critical for a multitude of more commonplace and prosaic activities in humans and other animals. Many cognitive activities, ranging from problem solving to social interaction to motor control, can be seen as containing an element of inductive reasoning.

Introduction

Inductive reasoning is one of the most important and ubiquitous of all problem-solving activities, and its use by nonhuman animals is of great interest within fields of psychology and biology such as cognitive psychology, ethology, evolutionary biology, learning, and neuroscience. In its most elementary form, it seems that most animals exhibit some capacity for inductive reasoning. For instance, sea slugs respond to a stimulus inductively based on past experiences with it (e. g., by "concluding" that the stimulus can be ignored; a case of habituation). Similarly, the fear response of a rodent to a stimulus that was repeatedly and predictably presented in conjunction with a harmful outcome is to some extent the result of an inferential prediction (i.e., the rodent "concludes" by induction that a bright light always precedes a foot shock). And Pavlov's dogs salivated in response to the ring of a bell

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that always came before meal time (the dogs "concluded" by induction that bells are a signal for lunch.)

There exists, however, an old divide on the interpretation of the relevant data on inductive reasoning: Associationism holds that the data can be mostly explained by simple associative processes whereby *reflexes* are modified through experience, while rationalism holds that more complex mechanisms are involved, and the existence of inductive reasoning is required to better explain the data (see Rescorla (1988) for a more complete analysis of these contrasting views).

Traditional associationist models posit that an organism's learned response to a stimulus is represented by the strength of the relevant associations between stimuli. In this view, the animal adapts to its environment through the process of reflex modification, and that the animal need not make predictions or choose between potential responses. In this sense, reasoning would be unnecessary to explain the examples provided above. Typically, associationists have argued that abstract, rule-governed representations do not really exist and that animals' (and humans') behavior can be just as well explained in terms of more specific learned associations between task inputs and outputs. Hence (according to this view), the only thing researchers should be concerned with is the specification of what gets associated with what (easier said than done, but still a relatively straightforward goal). Due in part to the predominance of associationism in psychology during the last century, it was until recently a commonplace assertion that inductive reasoning was beyond the capacity of nonhuman animals. In the past few decades, however, there has been an accumulation of evidence for animal inductive reasoning.

While traditional associationist models assume that an animal's interaction with the environment is passive (the animal behaves by the automatic adjustment of strengths of association only in reaction to stimuli), rationalist models assume that the interaction is active – the animal actively probes the environment to make determinations about optimizing behavior (Gallistel 2003). In that framework, "figuring things out in nature" is guided by how useful a stimulus is in reducing the animal's uncertainty about the time of occurrence of some relevant event. This knowledge comes in part from a distribution of graded degrees of belief over a range of candidate hypotheses, or in other words, it depends on inductive reasoning. Rationalists believe that there are problem-specific computational mechanisms that evolved to inform animals' behavior (Gallistel 2000). In this light, inductive reasoning is seen as a set of algorithms devoted to particular cognitive processes, such as causal inferences, probabilistic inferences, optimization of strategies from sampling, and categorization/concept formation (Nisbett et al. 1983). Here we will focus on these complex rational solutions that are (mostly) agreed upon to be cases of inductive reasoning.

It should be noted that the higher-order cognition proposed by rationalists and the associative rules proposed by associationists are not necessarily mutually exclusive. De Houwer et al. (2016) propose that associative learning is best thought of as an effect (i.e., the impact of paired events on behavior) rather than a specific mental process (e. g., the formation of associations). In other words, learning and behavior are mediated by higherorder mental processes akin to problem-solving processes such as inductive reasoning. (For more on that topic, see De Houwer et al. 2016).

Causal and Probabilistic Inferences

Causal inference is the process of reasoning about a causal connection between events based on the co-occurrence of those events. As noted in the Introduction, there are many parallels between instrumental and classical conditioning phenomena in animals on the one hand and contingency assessment and causal judgments by humans on the other (see Shanks (2007) for a recent review). The scientific framework for thinking about causal inference emerged from the associationists' work on instrumental conditioning. Nevertheless, despite the extensive evidence that bottom-up processes such as instrumental and Pavlovian learning play a fundamental role in the acquisition of causal knowledge, there is also accumulating evidence for the involvement of top-down processes of causal induction in animals. In this light, the co-occurrence of events is not simply stimuli for an automatic association but in fact evidence for the animal's rational mechanisms to make inferences about the world.

Some interesting examples help illustrate the case that animals have the capacity for complex causal inference. Takagi et al. (2016) used an expectancy violation procedure in cats similar to what is used with human infants to probe their understanding of causal relations and physical laws. The study asked whether cats can use a causal rule to infer the presence of an unseen object on hearing the noise it made inside a container (Takagi et al. 2016). Cats were presented with either an object dropping out of an opaque container or no object dropping out after hearing either a rattling sound (by shaking the container with the object inside) or no sound (by shaking the empty container). The relation between the sound and the object matched with physical laws in half of the trials (congruent condition) and mismatched in the other half (incongruent condition). Inferring the presence of an unseen object from the noise was predicted to result in longer looking time in the incongruent condition (indicating "surprise," i.e., that the cats' expectancy had been violated). In addition, the cats were also able to predict and reach for an object appearance when the container was turned over. These results all suggest that cats have a causal understanding of auditory stimuli, which is indicative of their capacity for inductive reasoning.

In another example, Cheney et al. (1995) examined if baboons can understand cause-effect relations in the context of social interactions by using a playback experiment. Under natural conditions, dominant female baboons grunt repeatedly to subordinate mothers when attempting to interact with those mothers' infants. The subordinate mothers occasionally respond to these grunts by uttering submissive fear barks. So, in their study, Cheney et al. played causally inconsistent scenarios to the baboons in which a lower-ranking female apparently grunted to a higher-ranking female and the higher-ranking female apparently responded with fear barks. As an important control to rule out simple associative processes, baboons heard a sequence made causally consistent by the inclusion of grunts from a third female that was dominant to both of the others. The researchers found that baboons responded very differently to each scenario: in the causally consistent scenario, they either ignored the calls or looked briefly in the direction of the speaker, while in the causally inconsistent scenario, the baboons stared (seemingly surprised) for a long time. That suggests that baboons recognize by inductive reasoning the factors that cause one individual to give submissive vocalizations to another.

Even rats also seem capable of some degree of causal reasoning. In one study, Blaisdell et al. (2006) concluded rats are able to derive predictions of the outcomes of interventions after passive observational learning of different kinds of causal models. Those outcomes, according to the study, cannot be explained by associationist models but are instead consistent with inductive reasoning models. As a last example, it is important to note that causal reasoning is also critical for the ability to adapt an object for use as a tool, a skill that has been demonstrated by many animals.

The ability to infer causation also depends on the ability to estimate probabilities, as actions (causes) do not always lead to the outcome (expected effects). Animals often face circumstances in which the best choice of action is not certain. Because of this, the ability to reason about probabilities has ecological relevance for many species. Environmental cues may be ambiguous, and choices may be risky (for a review on the theoretical side of decision-making under uncertainty, see Trimmer et al. 2011). The use of inductive reasoning to estimate probabilities requires sampling, and for that an individual must be able to estimate proportions.

Multiple species of animals seem to be able to successfully make judgments on proportions. For example, when foraging, animals go to where more will be available by considering the amount of food in alternative locations and the number of other individuals feeding at these different locations (Rugani et al. 2015). Or in a social context, some primates show inequity aversion; they are able to judge their own effort and payoff *relative* to another individual's (Brosnan and de Waal 2003).

Probabilistic inference goes one step beyond the ability to compare proportions, because the subject also needs to understand the sampling part of the procedure; that is, they need to make inferences about the probable identity of items drawn from populations, based on the distribution of items in those populations. In fact, a study on capuchin monkeys shows that some individuals could make judgments about proportions but could not reason about probabilities (Tecwyn et al. 2017). Capuchins had to select between hidden single-item samples randomly drawn from two jars. In their first experiment, Tecwyn et al. familiarized subjects with the single-item sampling procedure and determined their baseline performance in this task with two populations of items, where each jar contained only preferred or nonpreferred items (100% preferred vs. 100% nonpreferred), a task that does not involve any probabilistic judgment. In the other experiments, the researchers investigated the ability of the subjects to make inferences about random samples drawn from mixed pools of items in the two jars. Their results revealed that capuchins' performance suffered when the jars had mixed pools, because it required the individuals to reason probabilistically. Some individuals were still able to infer correctly, but not all did so. As the authors later discuss, this suggests that probabilistic inference might also require other complex mental processes such as inhibitory control and working memory.

Probabilistic inferences are not only restricted to apes. Even mice, for example, possess at least a simpler form of that capability (Berkay et al. 2016) and can adaptively modulate their decisions based on their experienced probability of outcomes. Probabilistic inference also seems to play a critical role in animals' social behavior, including decisions made according to game theory (Crowley 2003).

Optimization of Strategies from Sampling

The capacity of sampling for probabilistic inference is also related to an inductive inference of integrating information together, such as when an animal navigates home by triangulating spatial cues or develops optimal foraging strategies by combining different food densities in each area. In other words, this inductive reasoning optimizes strategies and decision-making by deriving the "whole" from samples of the component parts.

Wass et al. (2012) studied a form of inductive reasoning for foraging in mice using a Binary Tree Maze, inspired by procedures developed in human decision analysis for identifying the most efficient strategies to reach a goal. The Binary Tree Maze is a decision tree that bifurcates (at decision points) into branches. Each decision point is a potential goal location, and the end of each branch terminates in two "leaves," each of which also contains potential goal locations, providing (in this example) a total of 14 potential goals (although only a random selection of goals were baited on any particular trial). In Wass' study, the mice's task was to navigate the maze so as to inspect every potential goal for a piece of food. While there are many possible search strategies (or paths) to visit every node in a decision tree, the vast majority of these paths would be inefficient due to unnecessary node crossings (in other words, they would involve unnecessary retracing of a path or crosses of a location that had already been explored). What distinguishes the Binary Tree Maze from a standard maze learning task is that no single path is "best"; many routes are equally efficient, and a mouse might perform errorlessly across trials yet not follow the same route on successive trials. The degree to which a mouse could comprehend the structure of the maze from successive experiences in it and implement that information from its current location is a reflection of inductive reasoning. In the study, Wass et al. found that in their initial exposures to the decision tree maze, the mice's pattern

of behavior suggested a disorganized random search. However, within six trials, the patterns of individual animals stabilized and remained stable. At the end of several days of testing, many mice were performing at optimal foraging efficiency, suggesting that the mice quickly came to appreciate the underlying structure of the maze and fix on a strategy for its solution. (However, some mice still performed poorly, which is indicative of wide variability in those mice's inductive reasoning.) Furthermore, Wass et al. (2012) also determined if mice were relying on rote paths through the maze or whether they were engaging in an active search of the maze (a requisite for inductive reasoning). To make this determination, each mouse was allowed to begin its exploration of the maze, and upon making its first entry into a second level branch, the adjacent branch was blocked by lowering a black guillotine door. Had a mouse been following a rote (but nominally efficient) path through the maze, this manipulation would have disrupted the utilization of that rote path. Even after this, mice were still able to perform at a high level of efficiency.

This process of optimization of search is also seen in animals as evolutionarily distant from mammals as honey bees. Naug and Arathi (2007) investigated possible sampling and decision rules that the foragers use to choose one option over another by presenting foragers with choice tests in a foraging arena. They showed that a large part of the sampling and decision-making process of a foraging honey bee can be explained by decomposing the choice behavior into dichotomous decision points and incorporating the cost of sampling. The results suggest that a honey bee forager, by using a few simple rules as part of a probabilistic inference process, is able to effectively deal with the complex task of successfully exploiting foraging patches that consist of dynamic and multiple options (Naug and Arathi 2007).

Categorization

When encountering a new object in one's surroundings, the ability to recognize the item as a member of a known category, such as a potential food item or predator, can be crucial for survival. This process involves inductive reasoning given that it requires derivation of general principles from specific, sample observations (extracting the relevant rule out of limited samples and multiple options). Categorization has been well documented empirically, and many animals possess that ability.

In the 1960s, Herrnstein and Loveland (1964) showed that pigeons could learn to peck for reinforcement whenever pictures of people appeared on a screen and not to peck whenever pictures without people were presented. Many similar demonstrations followed, where birds and mammals were trained to categorize diverse classes of natural items, from trees and water to other animal species (examples in D'Amato and Van Sant 1988; Roberts and Mazmanian 1988). In addition to natural categories, animals have also successfully classified objects that would have no evolutionary significance to them, such as "cars" and "chairs" (Bhatt et al. 1988), ruling out the possibility that categorization is only based on innate concepts.

Some interesting examples help illustrate the case that animals are able to form categories from sampling the environment. A study with chimpanzees tested the animals' abilities to categorize photographs of natural objects (Tanaka 2001). Chimpanzees were initially trained to match different color photographs of familiar objects from four possible categories. In training, all the comparison stimuli were from the same category in one condition and from different categories in another condition. For all subjects, training performance was consistently better for the "different category" than for the "same category" trials. In probe trials after training, the sample and positive comparison stimuli were different items from the same category, and the foils were selected from among the three other test categories. Remarkably, individual performance was above chance in probe trials, suggesting that categorization by chimpanzees may transcend perceptual resemblance. Furthermore, the researchers replicated these same results later using novel stimulus items from the same four categories. This study demonstrates that chimpanzees can group perceptually different exemplars within the same category and suggests that these animals formed conceptual representations of the categories.

In an analogous study, Range et al. (2008) presented dogs with a touch-screen testing procedure, which allowed them to test visual discrimination without the presence of social cueing (social cues from humans are a known to be a big confounding factor in cognitive/behavioral studies with dogs, as they coevolved to be very sensitive to our cues). They first trained dogs to differentiate between a set of dog pictures and an equally large set of landscape pictures. All subjects learned to discriminate between the two sets and showed successful transfer to novel pictures. Interestingly, presentation of pictures providing contradictive information (novel dog pictures mounted on familiar landscape pictures) did not disrupt performance, which suggests that the dogs made use of a category-based response rule with classification being coupled to category-relevant features (of the dog) rather than to item-specific features (of the background).

Categorization is not only restricted to visual patterns; of course, it is just that those stimuli are easier for humans to study. In an investigation of categorization of song notes in great tits (Weary 1990), the birds were trained first to discriminate between two synthetic song notes. These sounds were models of naturally occurring song notes and differed from one another in five acoustic parameters. Once the birds learnt to discriminate between the two training notes, they were tested with notes that presented all combinations of the five parameters. Responses to these sounds showed that great tits relied almost exclusively on note frequency to form categories; other parameters such as amplitude modulation, frequency modulation, and the interaction between frequency and amplitude were all used to a much lesser extent.

In light of the abovementioned studies, one might wonder: How is inductive reasoning employed during the formation of categories? There are two main theories on how categorization is formed: exemplar theory and prototype theory. For decades, the exemplar theory was dominant in the categorization literature, and it assumes that there is one representational/processing system that serves all needs of categorization. According to the theory, organisms store exemplars as separate, individuated memory traces, refer new items to these stored exemplars, and include them in the category if they are similar enough. However, new research has shown that animals have prior expectations of prototypes (or family resemblance or natural kinds) and exclude exceptional cases (such as a weird looking predator exemplar from their list of that predator). In other words, inductive reasoning does not do the work from scratch in forming categories as expected by the exemplar theory. Instead, animals use inductive reasoning (among other processes) to infer coherent family resemblance based on prototypical categories already in place. In a review piece, Smith et al. (2016) argue that, based on converging evidence from multiple species, exemplar processing is insufficient to account for the available data on categorization. They claim that prototype averaging might be less effective and create errors of categorization in lab experiments (since in hypothetical situations, you can always imagine weird exemplars) but that in nature prototyping is good enough for a world with so many similarities within categories (most eagles look like eagles, most fruit trees look like fruit trees, etc.), and so being attuned to prototypes would be more advantageous for faster decisions. Or as Smith et al. (2016) writes: "familyresemblance categories dominate. Animals are adept at what they experience and what they need. Their adeptness is neither a coincidence nor a disability" (p. 273).

Conclusion

Inductive reasoning is present in many different animals, and the multiple ways in which it is expressed show continuities in nature: to different degrees and levels of competence, animals seem to be able to perform causal inference, probabilistic inference, optimizations of strategies based on observation, categorization, etc. Thus, animals are crucial behavioral ambassadors to the study of inductive reasoning in humans.

It is also important to keep in mind that although ubiquitous and important, inductive reasoning in practice can be riddled with problems, even in humans. We are far from the perfect, ideal case from which animals should be compared to, as we, in our daily/ecological (out of the armchair) lives, commit many mistakes in categorization and probabilistic inference (as shown by multiple studies that followed the seminal work of Kahneman and Tversky on how people overlook statistical variables such as sample size, correlation, and base rate when they solve inductive reasoning problems (Tversky and Kahneman 1974).

Cross-References

- Analogical Reasoning
- ► Categorization
- ► Causal Reasoning
- Deductive Reasoning
- Generalization
- ► Intelligence
- ► IQ
- ► Learning
- Working Memory

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